Barrier Curvature Dependent Transmission Probabilities for $^{16}O+^{12}C$, $^{16}O+^{58}Ni$, $^{16}O+^{154}Sm$ Systems

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Abstract

The transmission probabilities through the Coulomb barrier for reactions having difference barrier curvatures have been calculated. We have chosen three heavy-ion fusion reactions with a fixed projectile and three targets in different mass region, namely, $^{16}O+^{12}C$, $^{16}O+^{58}Ni$, $^{16}O+^{154}Sm$ systems. The transmission probability for each system is evaluated by using parabolic approximation of potential barrier. It is found that the variation of transmission probability depends on the different curvature of Coulomb barriers of colliding systems. For lighter systems, the transmission probability is closer to that of classical result. As the system becomes heavier, the curvature of the barrier becomes larger and consequently the deviation from the classical limit becomes significant. Thus, the quantum effect in transmission probability is found to be larger in heavy-ion fusion reactions with heavy target than those of lighter ones.

Keywords: Heavy-ion fusion, quantum tunneling, parabolic-approximation

Introduction

The quantum tunneling effect is a quantum phenomenon which occurs when particles move through a barrier that is higher in energy than the projectile energy. A classical particle will be reflected back from the potential barrier if its energy E is less than the barrier height V_0. According to quantum theory, there is a finite probability of the particle penetrating the potential barrier and being transmitted even if $E < V_0$. This is possible due to wave nature of particle and the particle appears to across the barrier without going over the top [1]. This phenomenon is known as “quantum tunneling effect”. Important examples of tunneling effect are alpha decay of nuclei, the cold emission or the tunneling of copper pairs between superconductors separated by a thin insulating layer. The aim of this work is to calculate transmission probability through Coulomb barriers in heavy-ion fusion reactions using parabolic approximation. We pay attention on the sensitivity of transmission probability on the curvature of the barrier.

Theoretical framework

We consider the tunneling phenomenon in heavy-ion fusion collisions at low energies. This process is a formation of a compound nucleus by combining two nuclei. For simplicity, the interaction potential between the projectile and the target is assumed as the sum of the Coulomb potential $V_C(r)$ and the nuclear potential$V_N(r)$ [2]. In low energy fusion process, the relative motion between the colliding nuclei has to overcome the barrier created by the strong cancellation between the long-range repulsive Coulomb force and the short-range attractive nuclear force. In this work, we use Akyuz-Winter form as nuclear potential [3]. The total interaction between the projectile and the target can be given by

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$$\mathrm{V(r) = V_C(r) + V_N(r).}$$

The Coulomb potential between two nuclei is given by $\mathrm{V_C(r) = \frac{Z_p Z_t e^2}{r}}$, where $\mathrm{Z_p}$ and $\mathrm{Z_t}$ are the atomic number of the projectile and target, r is the distance between the colliding nuclei and e is the elementary charge. The nuclear potential is taken to be the Akyuz-Winther form, which is defined by three parameters; the depth V_0, the radius R_0 and the diffuseness a and is given by;

$$\mathrm{V_N(r) = -\frac{V_0}{1 + \exp\frac{(r - R_0)}{\mathit{a}}}}.$$

The potential depth, V_0 is denoted by

$$\mathrm{V_0 = 16\pi \frac{R_1 R_2}{R_1 + R_2}} \gamma a$$

where $$a = \left[1/\left(1.17\left\{1 + 0.53\left(\mathrm{A_1^{-\frac{1}{3}} + A_2^{-\frac{1}{3}}}\right)\right\}\right)\right]\mathrm{fm},$$

and $$\mathrm{R = R_1 + R_2},$$

where the radius $\mathrm{R_i}$ has the form, $\mathrm{R_i = 1.20A_i^{\frac{1}{3}} - 0.09}$ fm, where i = 1, 2. Then the surface energy coefficient γ is given by

$$\gamma = \gamma_0 \left[1 - \mathrm{k_s}\left(\frac{\mathrm{N_1 - Z_1}}{\mathrm{A_1}}\right)\left(\frac{\mathrm{N_2 - Z_2}}{\mathrm{A_2}}\right)\right]$$

where $\gamma_0 = 0.095\ \mathrm{MeV/fm^2}$ and $\mathrm{k_s} = 1.8$.

Fig. 1 (a) shows the Coulomb barrier between two colliding nuclei which is formed by cancelling between Coulomb and nuclear potential as a function of the relative distance. In this work, the Coulomb barrier is approximated by an inverted parabola.

$$\mathrm{V(r) \sim V_B - \frac{1}{2}\mu\omega^2(r - R_B)^2}$$

The parabolic barrier is characterized by the barrier position $\mathrm{R_B}$, barrier curvature $\hbar\omega$ and barrier height $\mathrm{V_B}$, respectively. The transmission probability through Coulomb barrier can be calculated by solving the Schrodinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \mathrm{V_B} - \frac{1}{2}\mu\omega^2(\mathrm{r} - \mathrm{R_B^2})^2\right)\Psi = \mathrm{E}\Psi$$

$$\text{Transmission probability} = \frac{1}{1 + \exp[2\pi(\mathrm{V_B} - \mathrm{E})/\hbar\omega]}$$

where $\hbar\omega$ is the curvature of the parabolic barrier [2]. This formula is an exact quantum result and holds for all energies, and thus also for $\mathrm{E = V_B}$, i.e. at the barrier top.

In order to calculate the transmission probability through Coulomb barrier, we have to know curvature of the potential near the top of the barrier. Since the Coulomb barrier is approximated by the parabolic barrier, the curvature of the barrier can be related to the second derivative of the potential near the barrier top. $\hbar\omega = \hbar\sqrt{\frac{|V''(\mathrm{R_B})|}{\mu}}$

Results and Discussion

Firstly the Coulomb barriers for three reactions, namely, $^{16}O + {}^{12}C$, $^{16}O + {}^{58}Ni$ and $^{16}O + {}^{154}Sm$ systems have been calculated. Projectiles are fixed for all systems and targets are chosen from light, medium and heavy nuclei. We, then, calculate the barrier curvature and transmission probability for each system. Fig. 1 (b) compares the curvature of potential barriers for the systems considered. The red, green and blue lines represent the barriers for $^{16}O + {}^{12}C$, $^{16}O + {}^{58}Ni$ and $^{16}O + {}^{154}Sm$ systems, respectively. The values of the curvatures of the systems are also indicated. It is found that the curvature of heavier system is larger than that of lighter systems due to larger contribution of Coulomb interaction.

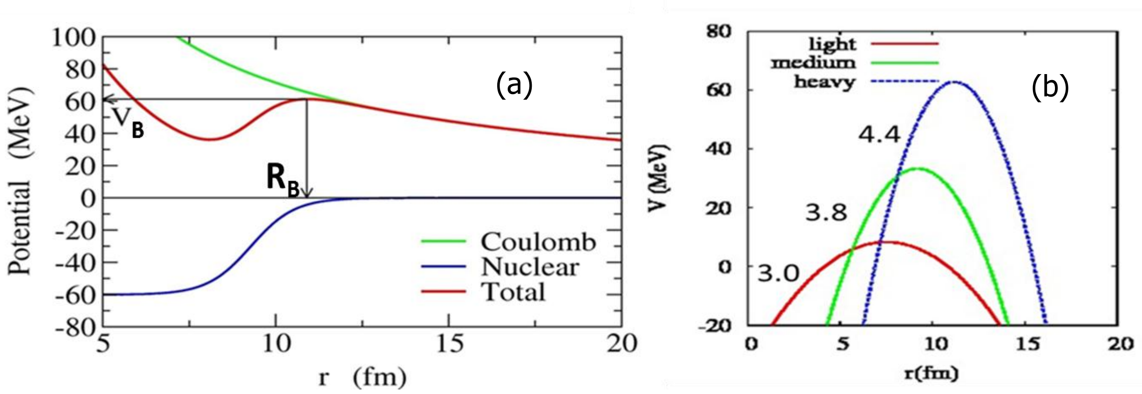

Fig. 1 (a) Typical Coulomb barrier between nuclei in Heavy-ion fusion reaction. (b) Coulomb barrier of the reactions with light (^{12}C), medium (^{58}Ni), and heavy (^{154}Sm) targets. The corresponding values of curvature of each barrier are also shown.

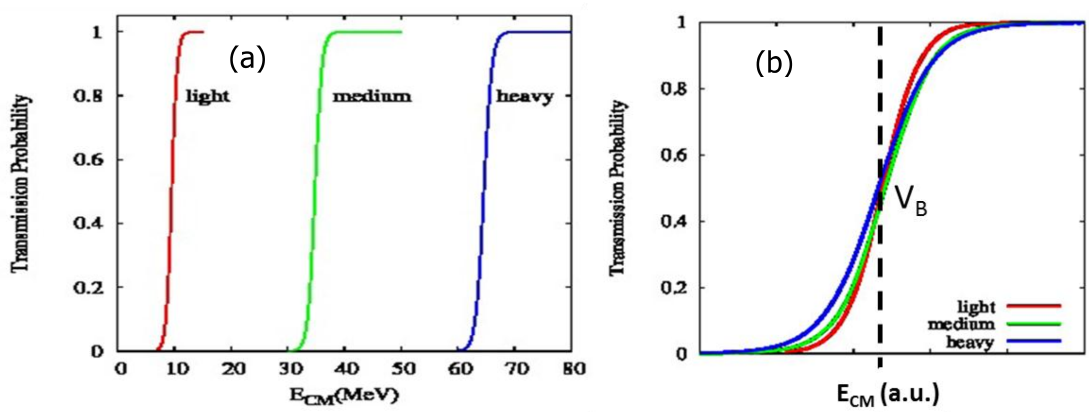

Fig. 2 (a) Transmission probabilities of ^{16}O+ light(^{12}C), medium(^{58}Ni), and heavy(^{154}Sm) targets as a function of incident energy. (b) The same as (a) but the energy values are shifted so that the Coulomb barrier heights coincide at V_B. Dashed line represents classical transmission probability.

Fig. 2 (a) shows the transmission probability versus energy for light to heavy nuclei. In Fig. 2 (b), the energy values are shifted so that the barrier heights of three systems coincide at a point. One can find that the variation of transmission probability is sensitive on the different curvature of Coulomb barriers of colliding systems. The transmission probability through barrier with smaller curvature goes faster to classical limit ie., the slope is much steeper for the lighter target compared to that for heavier one. As the system becomes heavier, the curvature of the barrier becomes larger and consequently the deviation from the classical limit becomes more significant. Thus, we can conclude that the quantum effect in transmission probability is found to be larger in heavy-ion fusion reactions with heavy target than those of lighter target.

Conclusion

In summary, we calculated the transmission probabilities through the Coulomb barrier for reactions having difference curvatures. We have chosen three reactions, namely, $^{16}O+^{12}C$, $^{16}O+^{58}Ni$, $^{16}O+^{154}Sm$ systems and calculate the curvature of the Coulomb barrier by using parabolic approximation. It is found that when the incident energy is less than the Coulomb barrier heights, the transmission probabilities through barriers having larger curvature are greater than those with smaller ones. In contract, if the energy of incident particles is greater than Coulomb barrier, transmission through the barrier with small curvature increases more rapidly to unity than those with large curvature. The results suggest that systems with larger curvature have larger quantum effects than those of smaller one. Larger quantum effect increases the tunneling probability below the barrier, and in the same sense, larger quantum effect increases the reflection probability above the barrier.

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